Accurate Inference of Termination Conditions

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Termination Analysis

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```
while (n > 1) {
    if (n % 2 == 0)
        n = n / 2;
    else
        n = 3 * n + 1;
}
```

Even for this simple loop, whether it terminates is still undetermined to this day.

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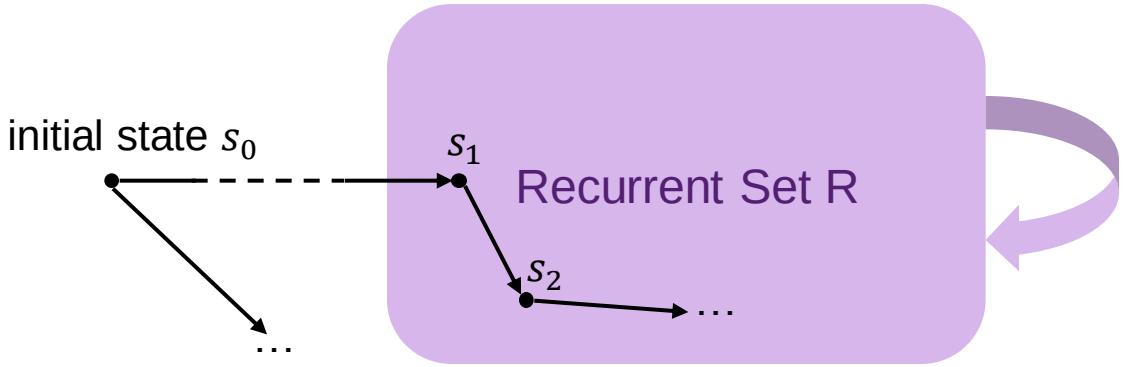
A recurrent set is a set of states R satisfying three conditions:

1. R is reachable from an initial state;
2. Any state in R has a successor;
3. For every state s in R, the successor of s remains in R.



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    if (x > 5)
        x = x + 1;
    else
        x = x - 1;
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This program terminates when $x \leq 5$

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A formula φ is a **termination condition** of a program P if P terminates under the condition φ.

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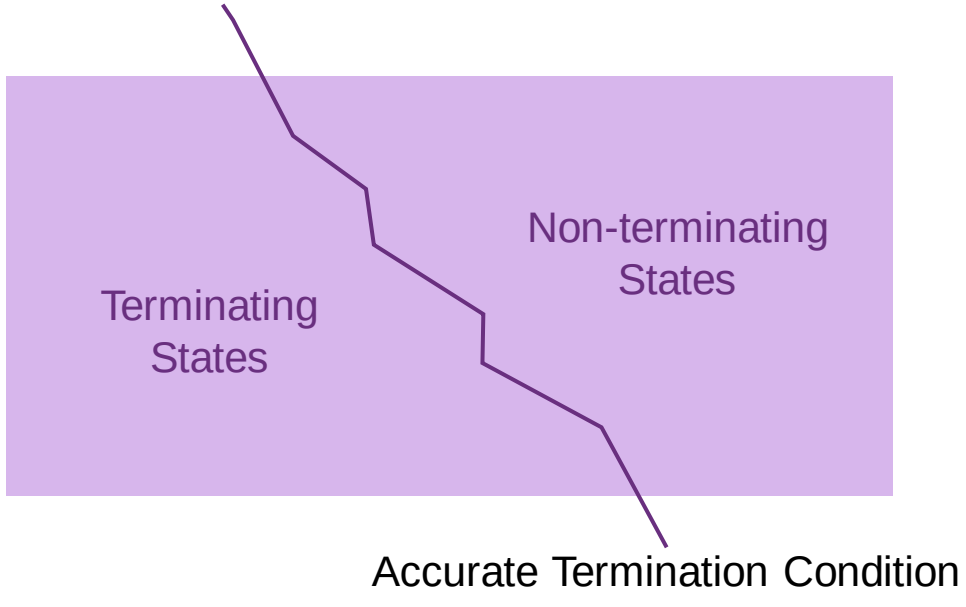
A formula φ is an **accurate termination condition** of a program P if P always terminates under φ and always does not terminate under $\neg\varphi$.

State-of-the-art

- Methods focusing on conditional termination remain limited.
  - Some methods[1] calculate the precondition that making a termination argument valid;
  - Some over-approximate non-terminating states[2] or under-approximate terminating states[3].
- Existing methods do not guarantee the precision of generated conditions.

- [1] Byron Cook, Sumit Gulwani, Tal Lev-Ami, Andrey Rybalchenko, and Mooly Sagiv. Proving Conditional Termination. CAV 2008.
- [2] Marius Bozga, Radu Iosif, and Filip Konečný. Deciding Conditional Termination. TACAS 2012.
- [3] Pierre Ganty and Samir Genaim. Proving Termination Starting from the End. CAV 2013.



Basic Idea

Program States

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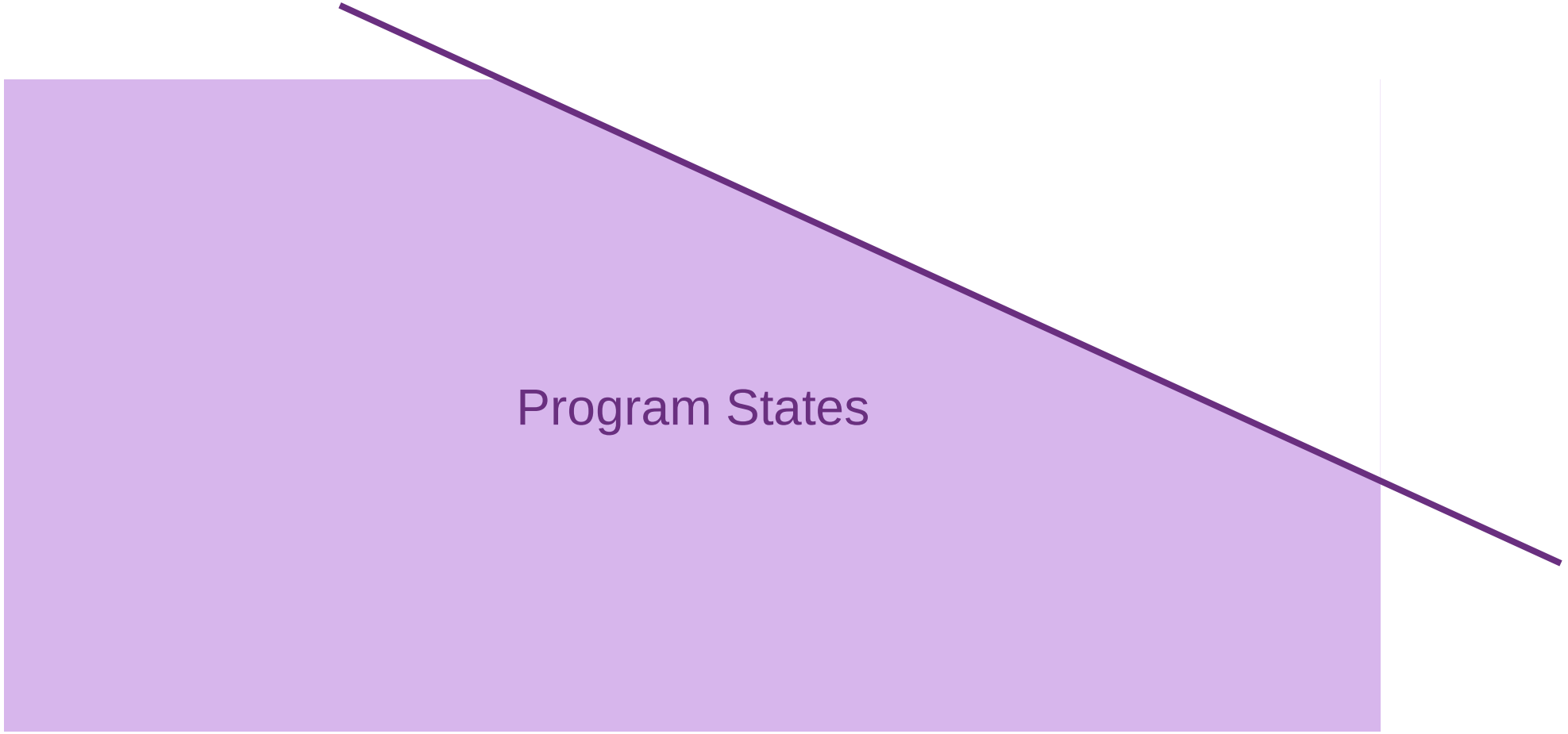

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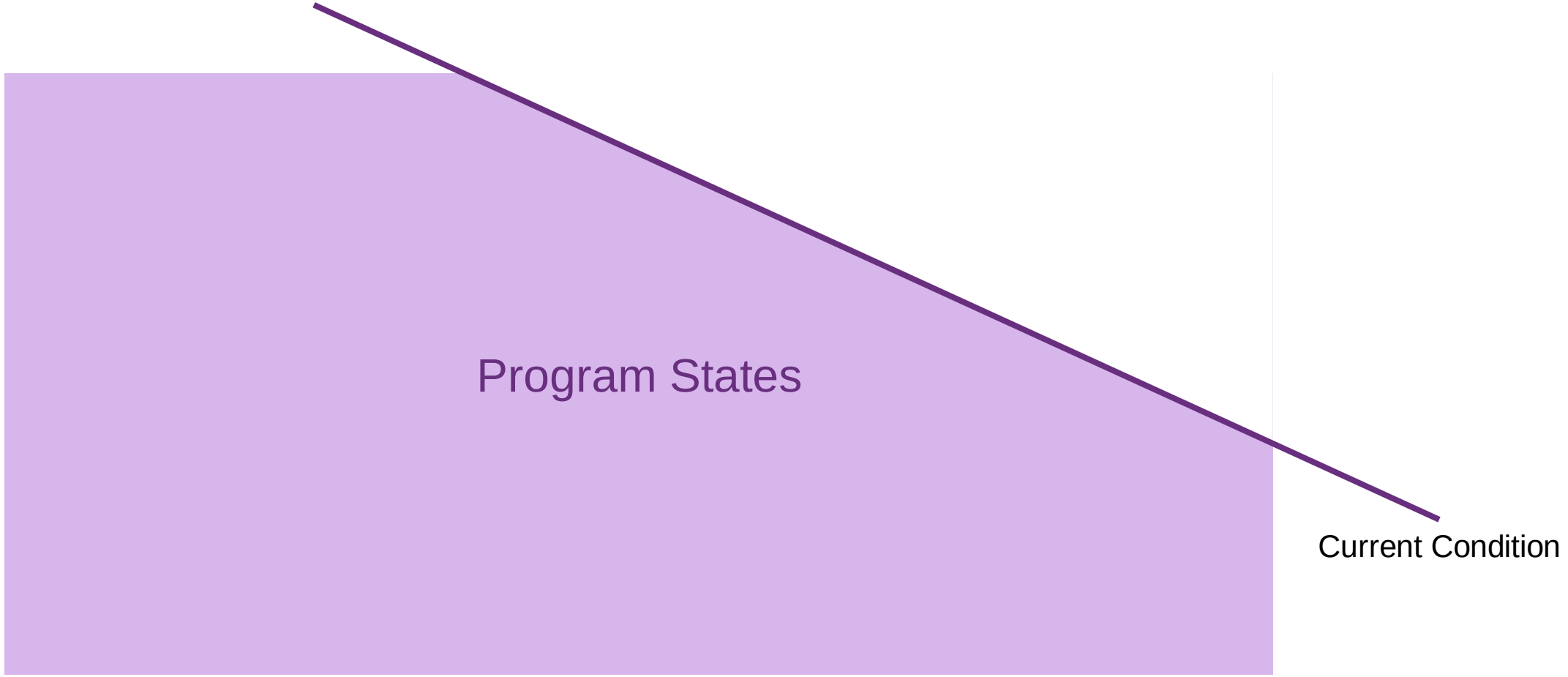

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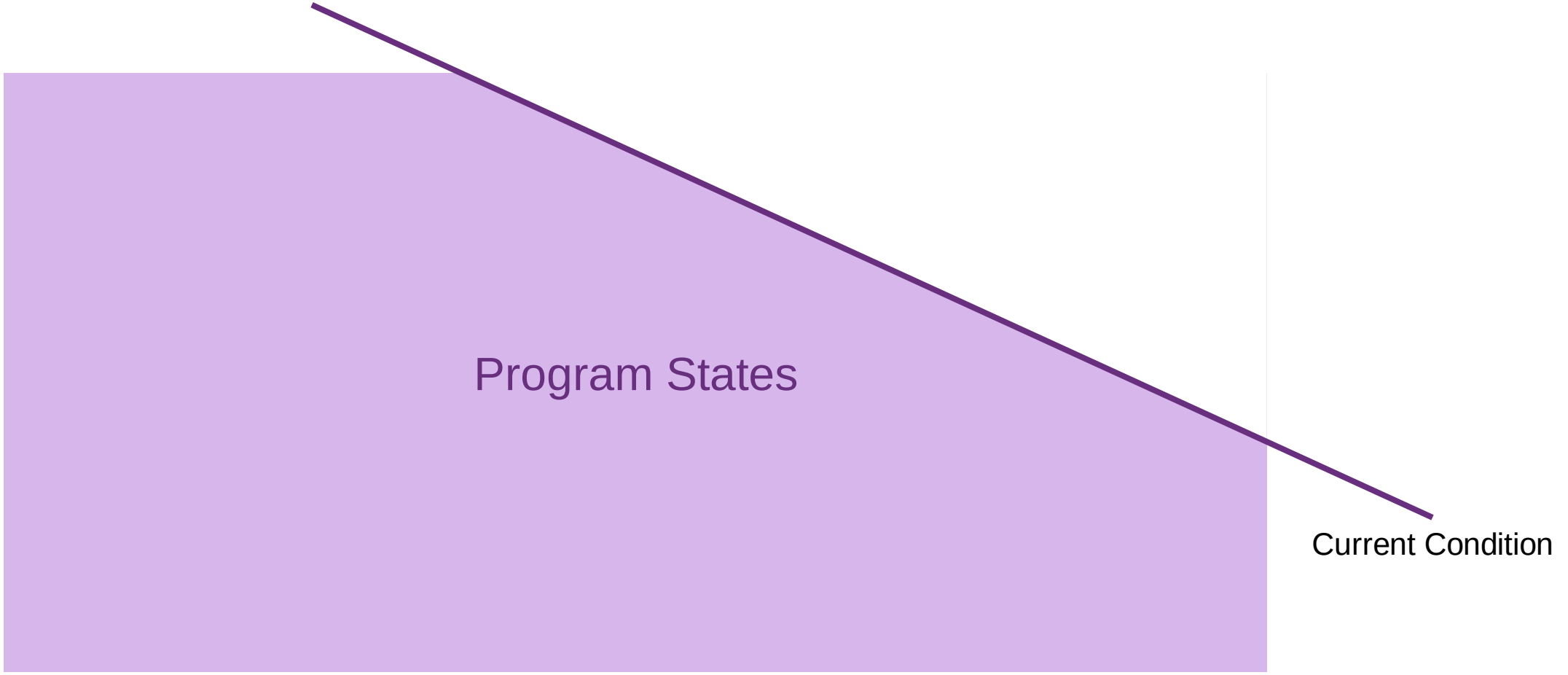

Is this program terminating?

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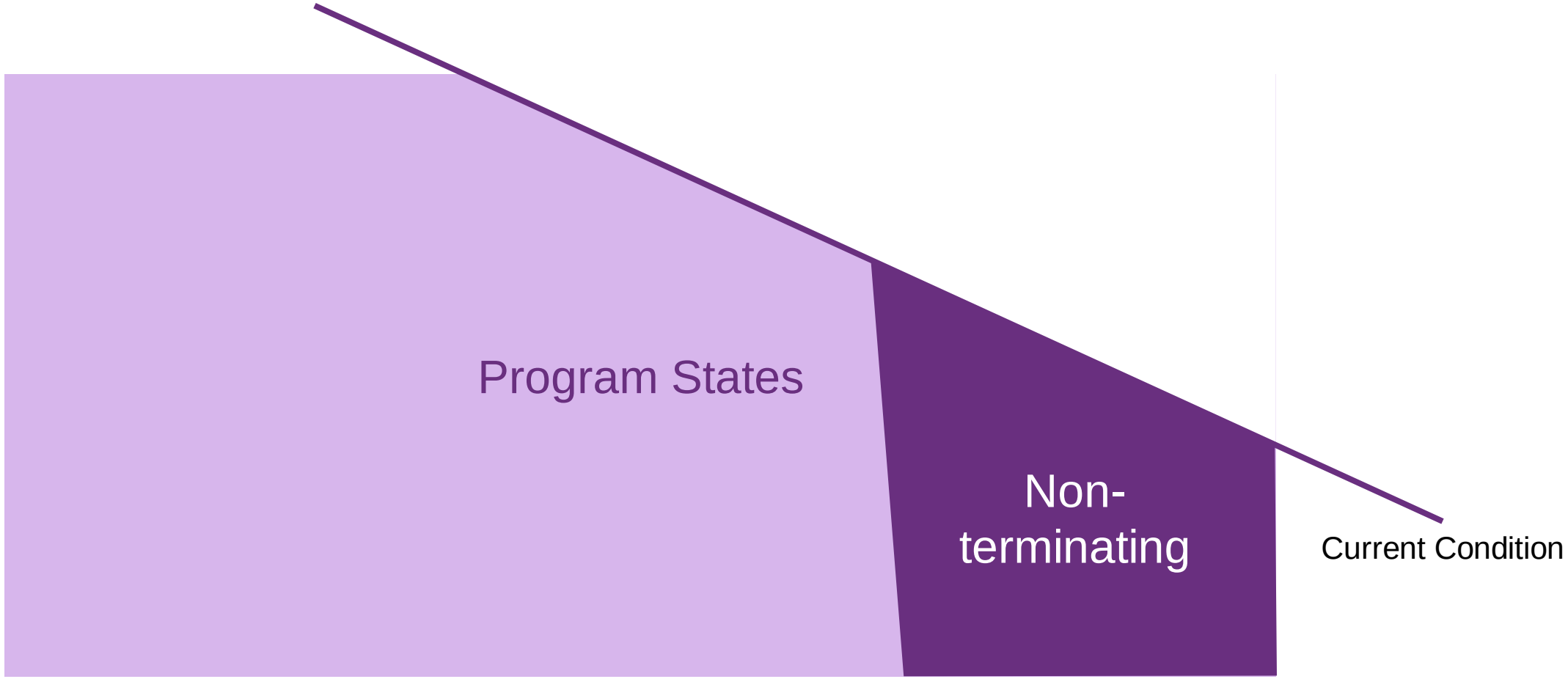

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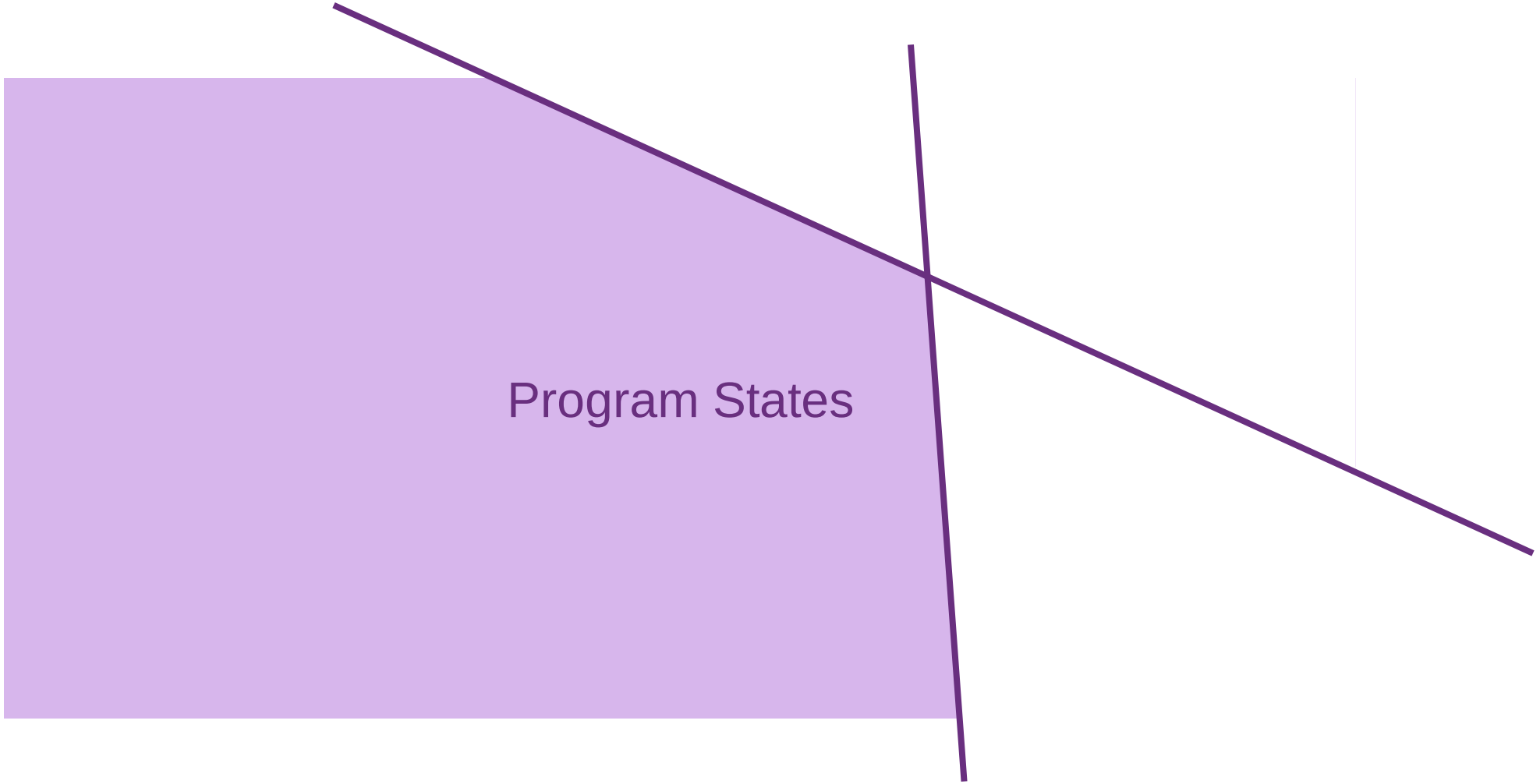

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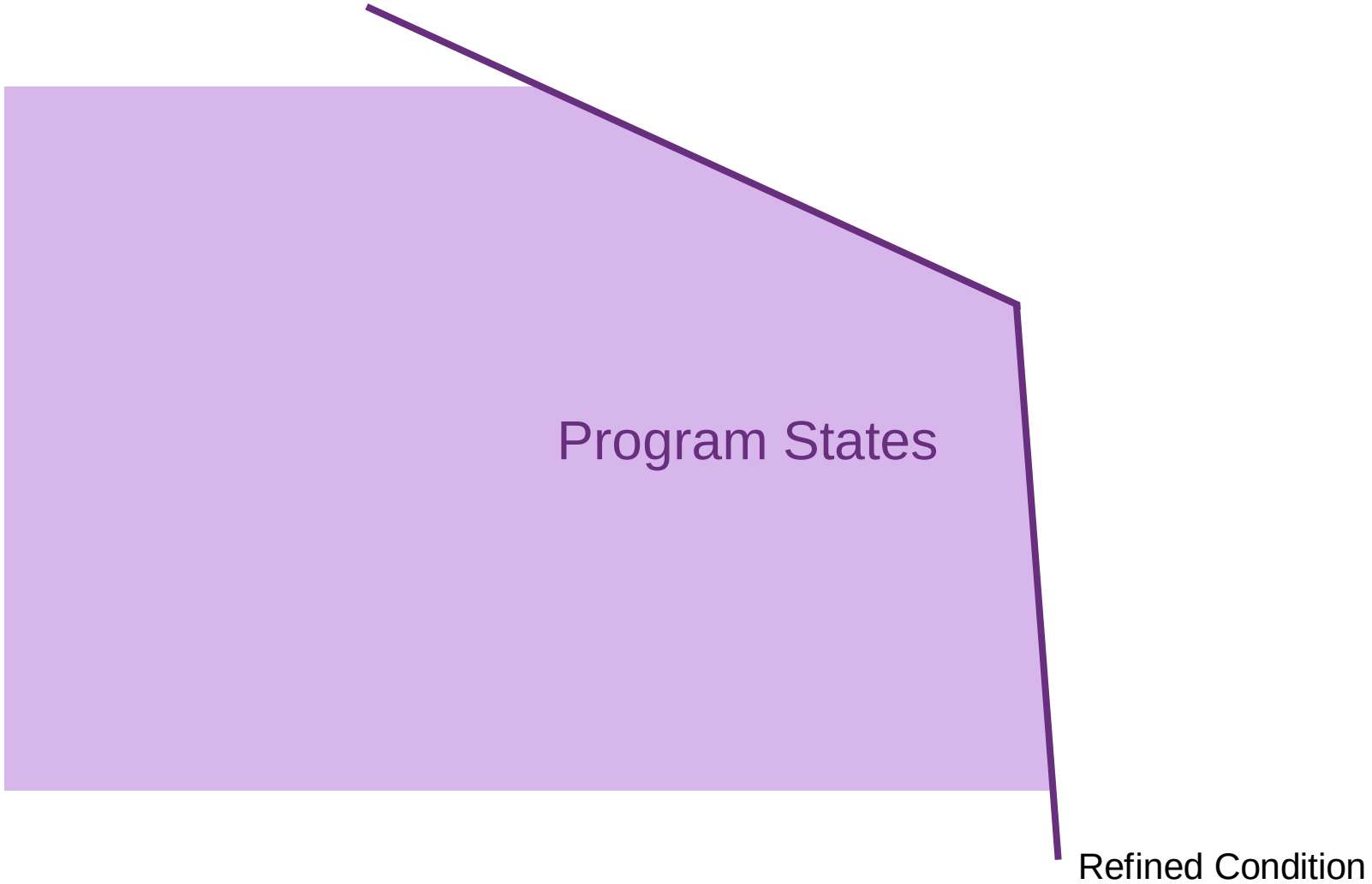

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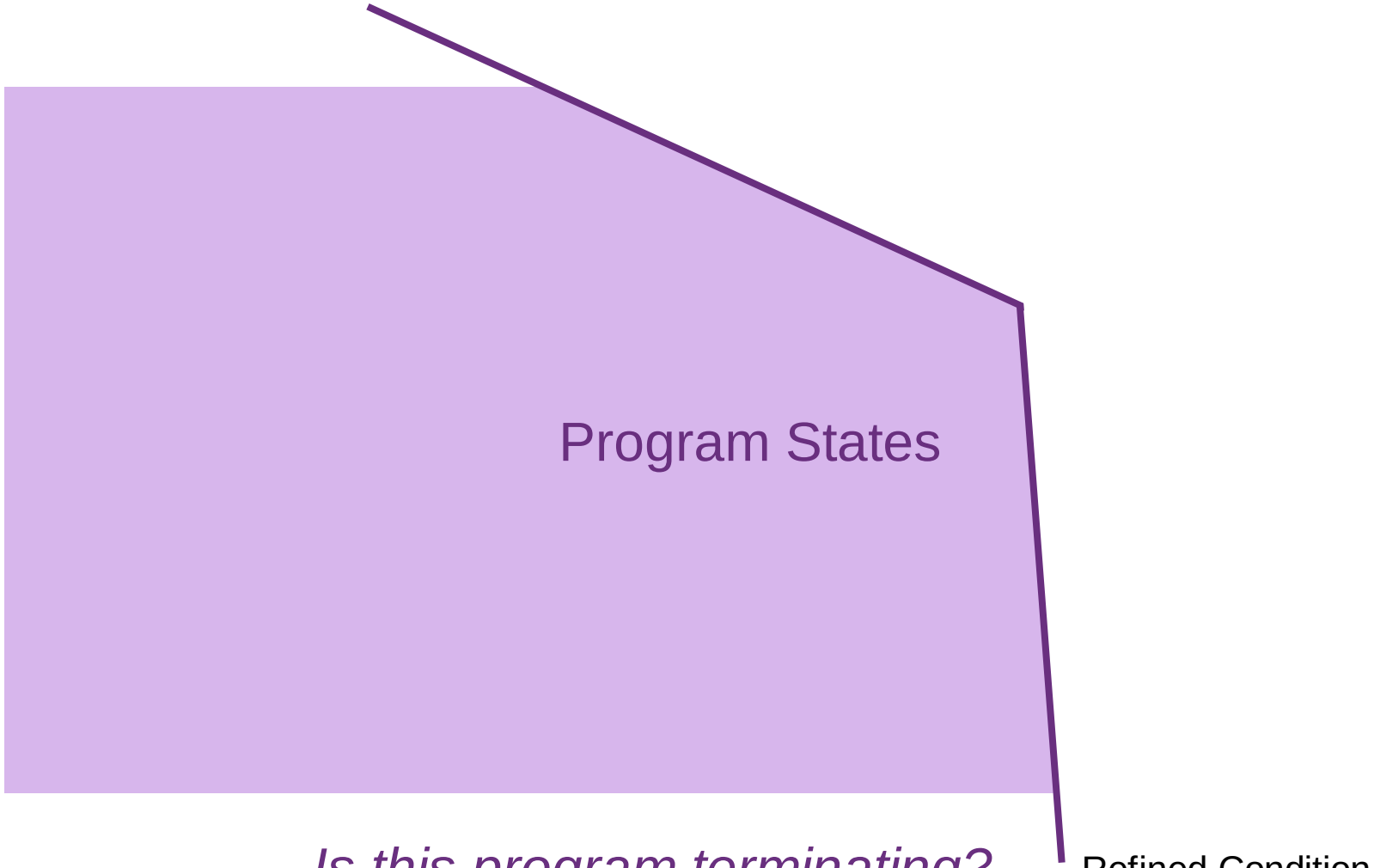

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Accurate termination condition: $-5 \leq t \leq 5$

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- Get another recurrent set $R_2 : t > 5$.

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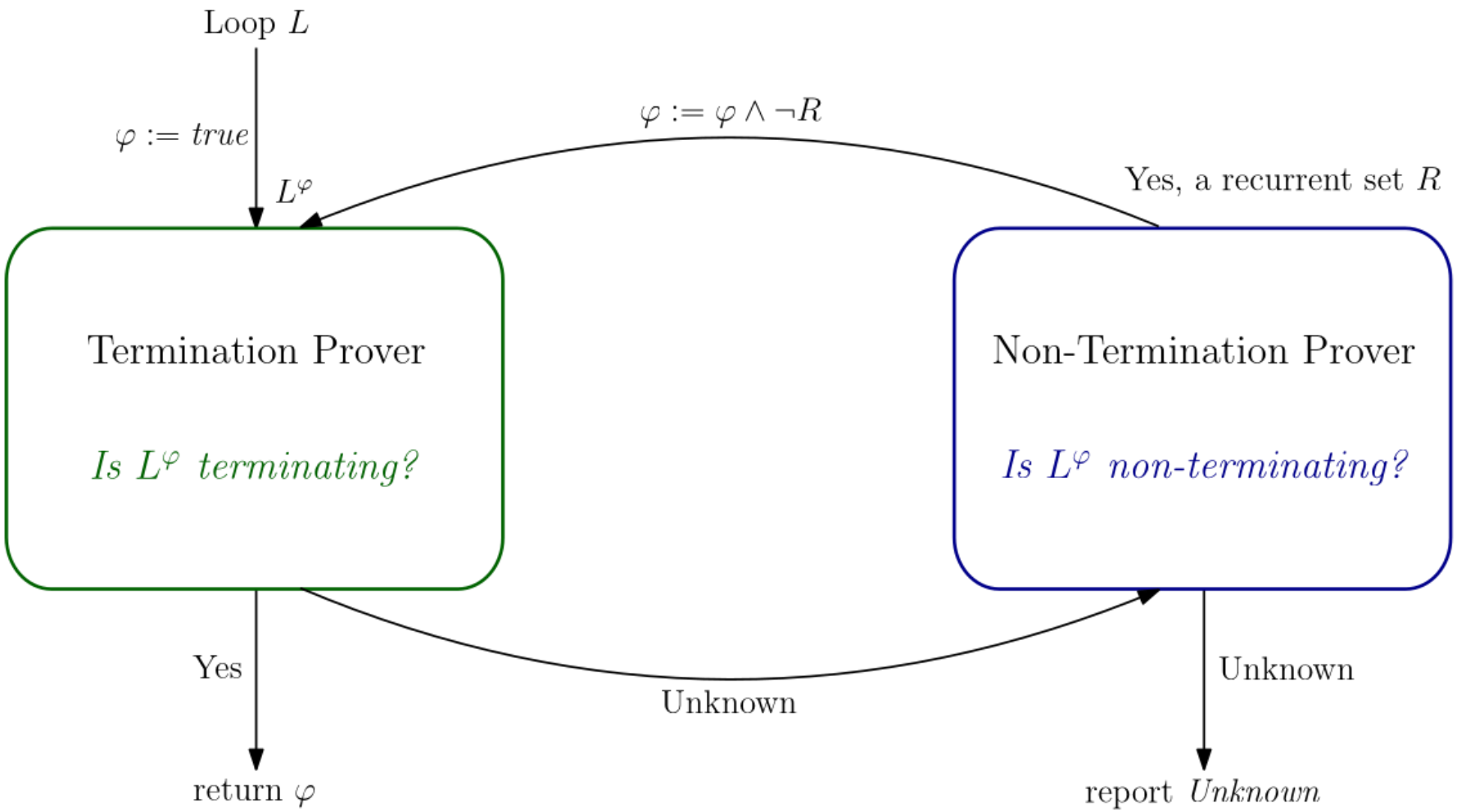
- Conclude that φ_2 is the accurate termination condition of the program.

Overview: Framework



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- A single witness suffices to establish non-termination, so provers often return a small recurrent set.
- However, to infer the accurate termination condition, all non-terminating states must be excluded.
  - An insufficient recurrent set can lead to an increased number of iterations and thus reduce overall efficiency.

Recurrent Set Generalization

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Recurrent Set Generalization

- Each loop is formalized using transition formulas:
  - Loop guard $\mathcal{G}(t)$: $t \neq 0$
  - Loop body $\mathcal{L}(t, t')$: $(t < -5 \rightarrow t' = 1 - t) \wedge$
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⇕

$\exists t, t'. \neg\Psi(t, t')$ is unsatisfiable

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Recurrent Set Generalization

- Conduct two generalization techniques based on the minimal UNSAT core.

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- Obtain recurrent set R_1: $t < -10 \wedge t < -8$.

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- **Constant generalization**: generalize $t < -8$ to $t < -5$.

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- Obtain recurrent set $R_1: t < -10 \wedge t < -8$.
- **Predicate elimination**: eliminate the redundant atomic predicate $t < -10$.
- Typically, an atomic predicate may be non-removable, but its range can be generalized.
- **Constant generalization**: generalize $t < -8$ to $t < -5$.
- Generalized recurrent set $R_1': t < -5$.

(Non-)Termination Provers

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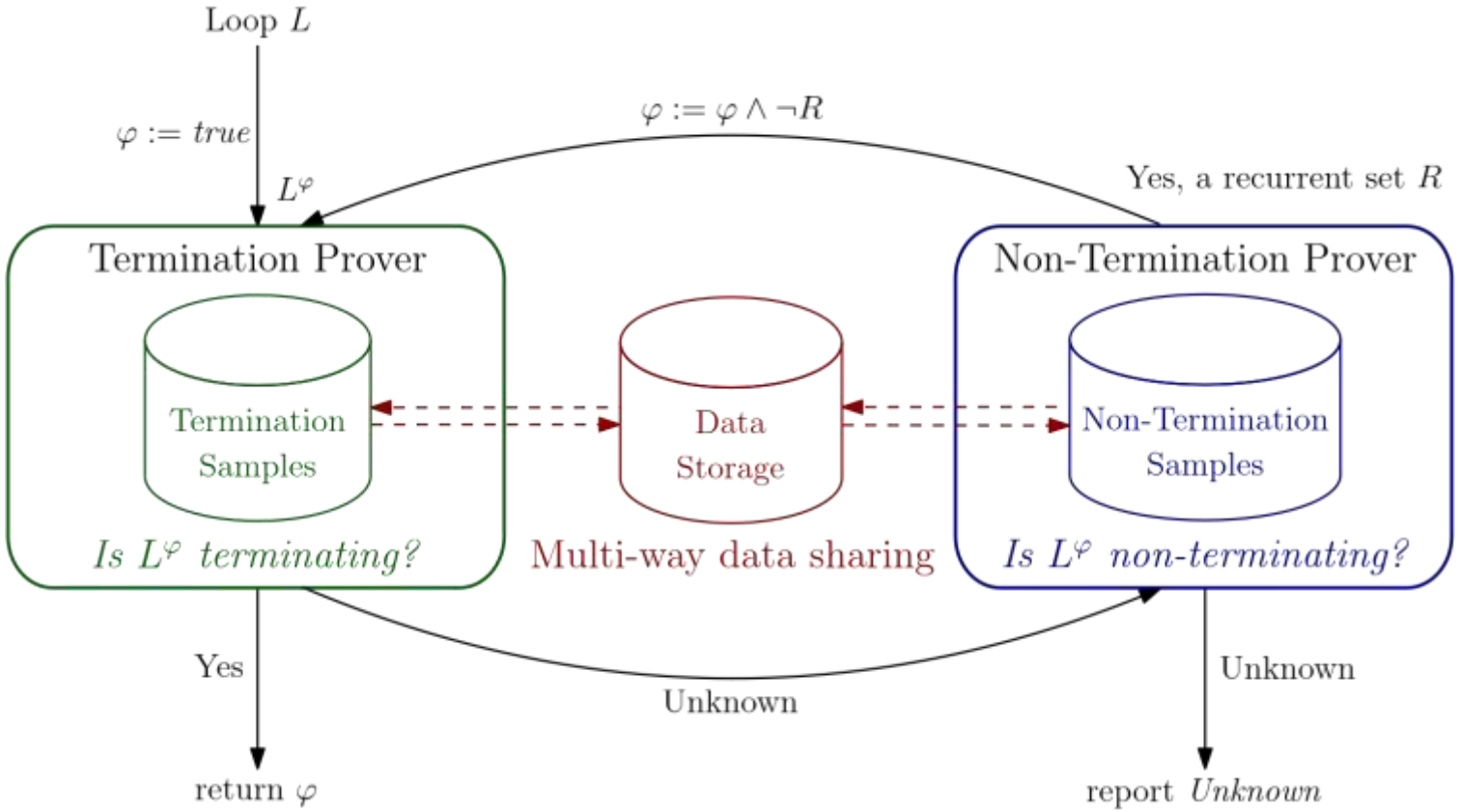
- We adopt **data-driven** provers, which use black-box method to synthesize arguments.

(Non-)Termination Provers

- We adopt **data-driven** provers, which use black-box method to synthesize arguments.
- Since each invocation of a prover targets a similar program, we design a **multi-way data sharing** mechanism, storing samples in each learning process for reuse.



Correctness

- Our method ensures soundness, accurateness, termination, and relative completeness.

Theorem 1 (Soundness)

For any loop L, if our algorithm returns a predicate φ, then φ is guaranteed to be a termination condition of L, provided that the termination prover used in the algorithm is sound.

Theorem 2 (Accurateness)

For any loop L, if our algorithm returns a predicate φ, then φ is guaranteed to be an accurate termination condition of L, provided that the termination and non-termination provers used in the algorithm are sound.

Correctness

Theorem 3 (Termination)

Given a loop L and a set of attributes $A = \{a_1, \ldots, a_n\}$, if the accurate termination condition φ^* can be expressed as a boolean combination of atomic predicates $a \leq c$ where $a \in A$ and c is any integer, then our algorithm is guaranteed to terminate, provided that the termination prover is sound, the non-termination prover is both sound and relatively complete, and the recurrent set identified in each iteration satisfies a specific assumption.

Theorem 4 (Relative Completeness)

Building upon the conditions of Theorem 3, if the termination prover is further complete, then the method is guaranteed to terminate with an accurate termination condition.

Evaluation

- We implement our tool as a prototype called CondTerm on top of data-driven provers ddlTerm[1] and RSLearn[2].
- **Benchmarks**: 170 terminating programs and 96 non-terminating programs.
- **Baseline**: Acabar[3], the latest tool with the same goal of synthesizing termination conditions as comprehensively as possible.

[1] Rongchen Xu, Jianhui Chen, and Fei He. Data-driven loop bound learning for termination analysis. ICSE 2022.
[2] Zhilei Han and Fei He. Data-Driven Recurrent Set Learning For Non-Termination Analysis. ICSE 2023.
[3] Pierre Ganty and Samir Genaim. Proving Termination Starting from the End. CAV 2013.

Evaluation

- Our tool solves more cases and generate more accurate termination conditions for both non-terminating and terminating benchmarks.

Benchmarks	NT (96 total)		T (170 total)	
Tools	CondTerm	Acabar	CondTerm	Acabar
#Solved.	**78**	51	**136**	64
#Accurate Sol.	**78**	41	**136**	56
Avg. T. on Sol.(s)	8.01	**0.81**	14.89	**3.57**

Evaluation

- The recurrent set generalization (R.S.G) and multi-way data sharing (M.D.S) techniques improve both efficiency and the number of solved cases.

Benchmarks	NT	
Enable R.S.G	Y	N
#Solved.	**78**	76(-2)
Overall Avg. T.(s)	**39.11**	54.86(40.3%)
Overall Avg.Iter.	**3.03**	3.53(16.5%)

Benchmarks	NT		T	
Enable M.D.S	Y	N	Y	N
#Solved.	**78**	78(0)	136	134(-2)
Avg. T. on Both Sol.(s)	**8.01**	8.62(7.6%)	14.03	14.44(2.9%)
Avg.Iter. On Both Sol.	**2.29**	2.31(1.3%)	1	1

Thanks!